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INTERACTION OF TWO NONIDENTICAL CONVERGING SKEW SHOCK WAVES IN A GAS WITH A CONSTANT POLYTROPIC INDEX

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Consideration is given to the interaction of two skew shock waves (SSW) moving at different velocities in opposite directions to each other in a gas with a constant polytropic index k. An analysis is made of the changes in configurations of the shock waves due to their interaction at round angles of contact within the range 0 – π.

Introduction. The problem of interaction of plane shock waves (SWs) in a gas is similar in many respects to the process of SSW refraction on the interface of media of different density [1]. This is seen well in a coordinate system superposed with the point at which the SW fronts are immovable. With SSW incidence on the interface of two media it was the point of contact (PC) of the SSW with the interface, while for the interaction of two plane SSW in a section perpendicular to their line of contact it will be the point O (PC) (Fig. 1a). The angles made by the fronts of both SSWs with the straight line along which the PC (SC) moves are given by the relations

$$\frac{D_H}{\sin\varphi_H} = \frac{D_L}{\sin\varphi_L}, \quad \varphi = \varphi_H + \varphi_L \tag{1}$$

for $\varphi \le \arccos(-1/a)$ (Fig. 1a) and

$$\frac{D_H}{\sin\alpha} = \frac{D_L}{\sin\varphi_L}, \quad \alpha = \pi - \varphi_H \tag{2}$$

for $\varphi > \arccos(-1/a)$ (Fig. 1b). For the sake of definiteness we assume that $D_H > D_L$ and $a = D_H/D_L > 1$. Expressing φ_H and φ_L in terms of the round angle made by the interacting SSW (4), we arrive at

$$\varphi_H = \begin{cases} \arctan\dfrac{a\sin\varphi}{1 + a\cos\varphi}, & \varphi \le \arccos\left(-\dfrac{1}{a}\right), \\ \pi - \arctan\dfrac{a\sin(\pi-\varphi)}{a\cos(\pi-\varphi) - 1}, & \varphi > \arccos\left(-\dfrac{1}{a}\right), \end{cases}$$
$$\varphi_L = \begin{cases} \arctan\dfrac{\sin\varphi}{a + \cos\varphi}, & \varphi \le \arccos\left(-\dfrac{1}{a}\right), \\ \arctan\dfrac{\sin(\pi-\varphi)}{a - \cos(\pi-\varphi)}, & \varphi > \arccos\left(-\dfrac{1}{a}\right). \end{cases} \tag{3}$$

Plots of the changing angles φ_H and φ_L for different a are given in Figs. 2a and b. The value of φ_H is a monotonic increasing function of φ, while φ_L increases up to its limiting value $\varphi_L = \varphi_t - \pi/2$ (at this point $\varphi_H \equiv \pi/2$), where

$$\varphi_t = \arccos\left(-\frac{1}{a}\right), \tag{4}$$

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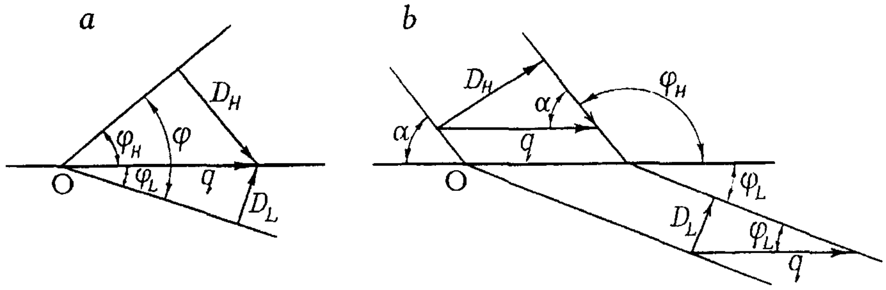


Fig. 1. Scheme of the collision of two SSW at: a) $\varphi \leq$ arccos $(-1/a)$; b) $\varphi >$ arccos $(-1/a)$.

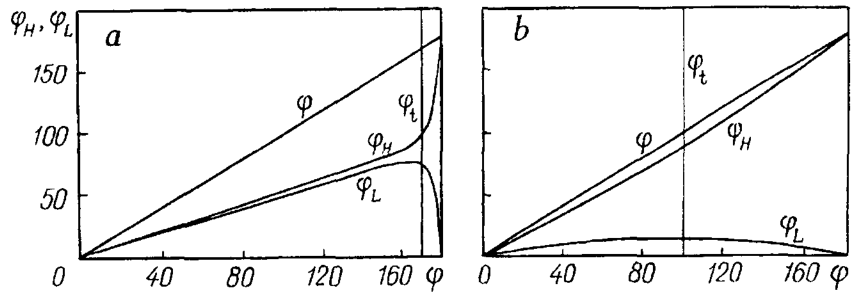


Fig. 2. Plots of changing the angles determining the branches H and L for: a) $a = 1.031$; b) $a = 5$.

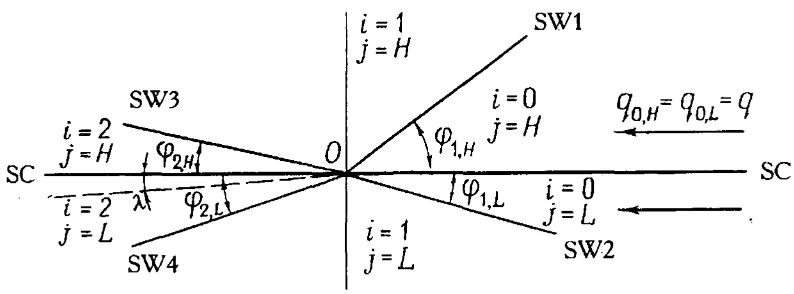


Fig. 3. Scheme of interaction of SSW in the RCR.

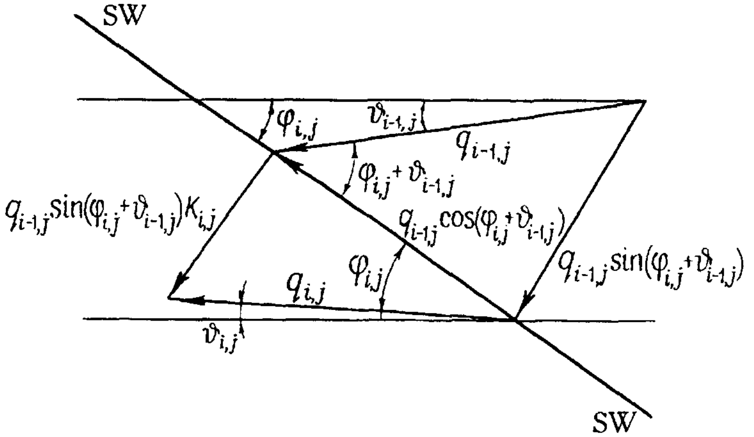
then it begins to decrease monotonically and at $\varphi = \pi$ vanishes. As is seen below, the angle φ_t plays an important part in forming a shock-wave configuration typical for the interaction of two SSW. From (4) it follows that φ_t is a constant of the interaction and does not depend on gas parameters.

Small Angles of Interaction. At small angles of interaction (Fig. 3) a regular collision regime (RCR) should be expected when immediately behind each initial SSW an SW appears which departs from the SC. Parameters of gas flow behind each discontinuity surface are as follows:

$$p_{i,j} = \frac{2\rho_{i-1,j}\, q_{i-1,j}^{2}}{k+1} \sin^2 (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j}) - \frac{k-1}{k+1} p_{i-1,j},$$

$$K_{i,j} = \frac{k + 1 + (k-1)\dfrac{p_{i,j}}{p_{i-1,j}}}{k - 1 + (k+1)\dfrac{p_{i,j}}{p_{i-1,j}}}, \quad \rho_{i,j} = \frac{\rho_{i-1,j}}{K_{i,j}}, \tag{5}$$

$$q_{i,j} = q_{i-1,j} \cos (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j}) \sqrt{1 + \tan^2 + (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})\, K_{i,j}^2},$$



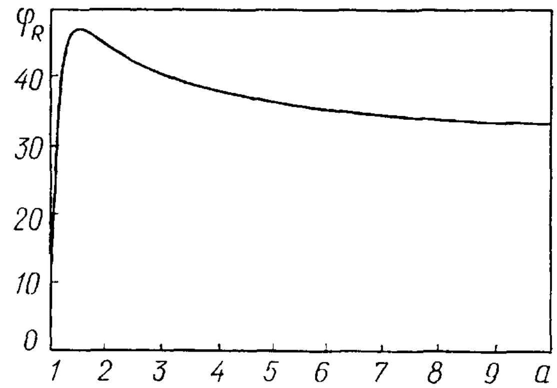


Fig. 4. Scheme of changing the velocity vector of the gas flow upon its passage across the discontinuity surface.

Fig. 5. Limiting angle of regular collision versus a.

$$\vartheta_{i,j} = \varphi_{i,j} - \arccos \sqrt{\left(\frac{1}{1 + \tan^2 + (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})\, K_{i,j}^2}\right)}\,, \quad s_{i,j} = \begin{cases} 1, & i = 2\,, \\ 0, & i \neq 2\,. \end{cases}$$

System (5) is universal for the entire plane with a corresponding change of $i = 1, 2$ and $j = H, L$. A scheme of decomposition of the velocity vector of the gas flow upon intersection of the SW front is shown in Fig. 4. Values in the region $i = 0$, $j = H$ and $i = 0$, $j = L$ coincide with the initial parameters of the gas

$$\rho_{0,H} = \rho_{0,L} = \rho\,, \quad p_{0,H} = p_{0,L} = p\,, \quad q_{0,H} = q_{0,L} = q = \frac{D_H}{\sin \varphi_{1,H}} = \frac{D_L}{\sin \varphi_{1,L}}\,, \tag{6}$$

while k remains unchanged throughout the volume. It should be noted that the condition of constancy of k simplifies calculations and does not introduce radical changes in the interaction picture. At $i = 1, 2$, $j = H, L$ combined system (5) consists of 16 equations with 18 unknown quantities, namely, $\rho_{1,H}$, $\rho_{2,H}$, $\rho_{1,L}$, $\rho_{2,L}$, $p_{1,H}$, $p_{2,H}$, $p_{1,L}$, $p_{2,L}$, $q_{1,H}$, $q_{2,H}$, $q_{1,L}$, $q_{2,L}$, $\vartheta_{1,H}$, $\vartheta_{2,H}$, $\vartheta_{1,L}$, $\vartheta_{2,L}$, $\varphi_{2,H}$, and $\varphi_{2,L}$, and is closed by the conditions of equality of the pressure and normal component of the velocity of the gas flow along the SC behind the PC (see Fig. 3):

$$p_{2,H} = p_{2,L}\,, \quad q_{2,H} \sin \vartheta_{2,H} = q_{2,L} \sin \vartheta_{2,L}\,. \tag{7}$$

A numerical investigation of (5) at conditions (7) reveals that a solution of the combined system exists for a series of small φ values. The dependence of the limiting critical angle $\varphi_R(a)$, which bounds the region of RCR values, is investigated in [2] and shown in Fig. 5. It has been established [2] that for gases φ_R is connected only with a and remains practically unchanged with changing k, D_H, D_L, and ρ. At $a \to 1$ $\varphi_R \to 0$, but at $a \to \infty$ $\varphi_R \to 29.3^\circ$, and for $a < 1.117$ an RCR is impossible [2]. At $\varphi > \varphi_R$ rotation of the velocity vector of the flow q toward the SC in the half-plane H cannot be already compensated by the corresponding rotation of the SC through the angle

$$\lambda = \arctan \frac{q_{2,H} \sin \vartheta_{2,H}}{q}\,, \tag{8}$$

that causes separation of the PC from the SC and emergence of a Mach wave between two triple points with the lower point on the SC (O) (Fig. 6) and the upper one (O) above the SC in the half-plane H. In this case, in system (5) the subscript j runs through four values: $j = H, L, M, N$, for the first two ($j = H, L$), $i = 1, 2$ and for $j = M$, N, $i = 1$, $p_{0,M} = p_{0,N} = p_{0,H} = p_{0,L} = p$, $q_{0,M} = q_{0,N} = q_{0,H} = q_{0,L} = q$, and $\rho_{0,M} = \rho_{0,N} = \rho_{0,H} = \rho_{0,L} = \rho$. The formed combined system of equations contains 24 relations and 28 unknown quantities: $\rho_{1,H}$, $\rho_{2,H}$, $\rho_{1,L}$, $\rho_{2,L}$, $\rho_{1,M}$, $\rho_{1,N}$, $p_{1,H}$, $p_{2,H}$, $p_{1,L}$, $p_{2,L}$, $p_{1,M}$, $p_{1,N}$, $q_{1,H}$, $q_{2,H}$, $q_{1,L}$, $q_{2,L}$, $q_{1,M}$, $q_{1,N}$, $\vartheta_{1,H}$, $\vartheta_{2,H}$, $\vartheta_{1,L}$, $\vartheta_{2,L}$, $\vartheta_{1,M}$, $\vartheta_{1,N}$, $\varphi_{2,H}$, $\varphi_{1,M}$,

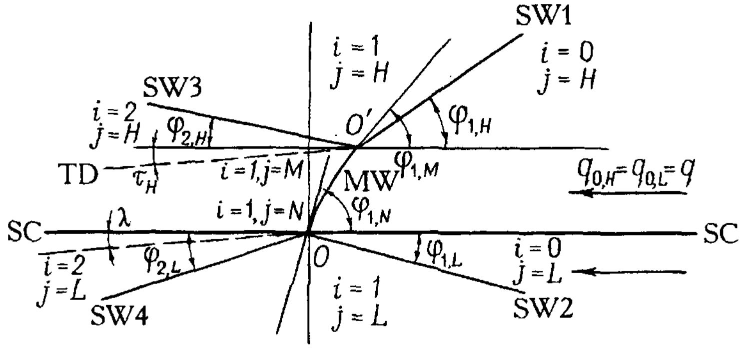


Fig. 6. Scheme of interaction of SSW in the IR1.

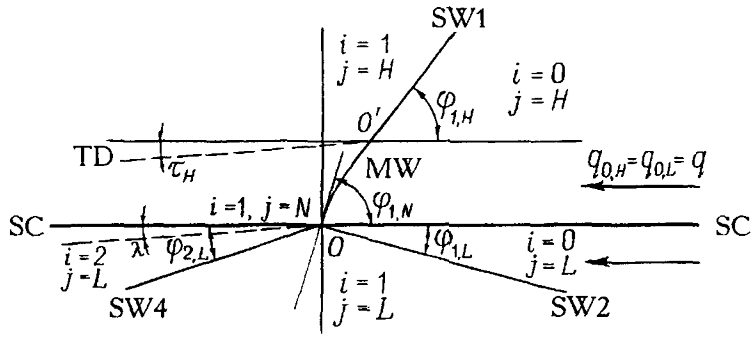


Fig. 7. Scheme of interaction of SSW in the IR2.

$\varphi_{1,N}$, and $\varphi_{2,L}$. The system can be closed by the conditions of equality of the pressure and the normal velocity component of the gas flow on both sides of the tangential discontinuity (TD) (Fig. 6):

$$p_{2,H} = p_{1,M}\,, \quad q_{2,H} \sin \vartheta_{2,H} = q_{1,M} \sin \vartheta_{1,M} \tag{9}$$

and the SC:

$$p_{2,L} = p_{1,N}\,, \quad q_{2,L} \sin \vartheta_{2,L} = q_{1,N} \sin \vartheta_{1,N}\,. \tag{10}$$

For this regime, referred to as the first irregular regime (IR1):

$$\tau_H = \arctan \frac{q_{1,M} \sin \vartheta_{1,M}}{q}\,, \quad \lambda = \arctan \frac{q_{1,N} \sin \vartheta_{1,N}}{q}\,. \tag{11}$$

A calculation shows that for strong SWs and large a the IR1 is implemented in air in the range of angles: $44^\circ \le \varphi \le 93^\circ$ at $a = 2$ and $39^\circ \le \varphi \le 76.6^\circ$ at $a = 5$. With decreasing a the upper angle of the IR1 decreases ($\sim 58.1^\circ$ at $a = 1.333$; $\sim 48.7^\circ$ at $a = 1.111$; and $\sim 47.9^\circ$ at $a = 1.031$. Here, for $a < 1.117$ the RCR is absent [2] and the IR1 is the initial regime of interaction). All data presented here have been calculated for air ($\rho = 1.29$ kg/m^3, $k = 1.4$); however, in the calculations for krypton (3.733 and 5/3) and methane (0.717 and 4/3) no crucial changes have been observed.

The following consideration of the interaction of strong SSW is convenient for division into three regions in relation to a: the region of large a, $a > 1/\sin \varphi_R$ (for strong SSW $\rho_R \to 29^\circ$ [2], $a > 1.9-2.0$); the region of moderate a, $1/\sin \varphi_c < a \le 1/\sin \varphi_R$ (for the majority of gases $\varphi_c \approx 60-68^\circ$, i.e., $1.95 < a \le 1.9$); and the region of small a, $a \le 1/\sin \varphi_c$ ($a \le 1.095$).

Irregular Regimes of Interaction for Large a. In this region an increase in φ is accompanied by the IR1 up to $\varphi_{1,H} = \varphi_c$ when the flow behind the SW1 front (Fig. 7) becomes subsonic and SW3 disappears. The value of φ_c for polytropic gases can be calculated exactly [1]. Here, i takes the value $i = 1, 2$ only for the lower half-plane ($j = L$), for all remaining j $i = 1$. The subscript j runs through the values $j = H, N, L$ since with disappearance of SW3 the regimes H and M coincide at point O'. The system obtained upon development of (5) consists of 16

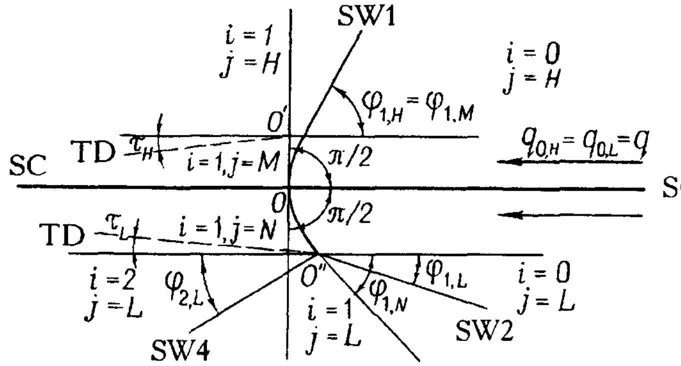


Fig. 8. Scheme of interaction of SSW in the IR3.

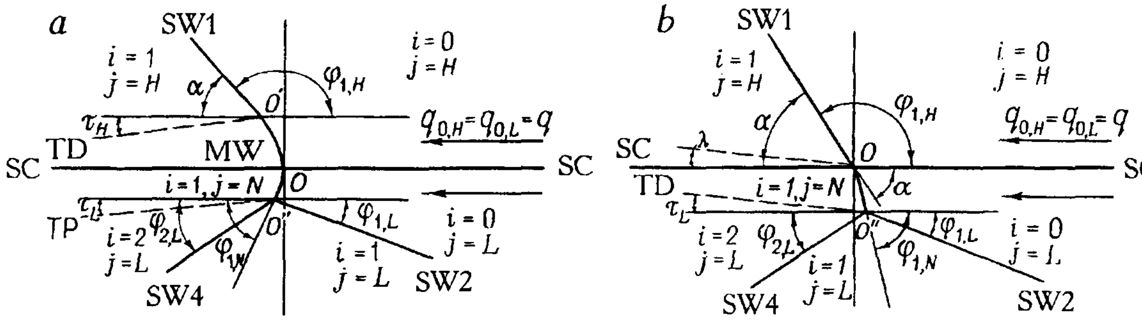


Fig. 9. Scheme of interaction of SSW in irregular regimes: a) IR1; b) IR4.

equations with 18 unknown quantities, namely, $\rho_{1,H}$, $\rho_{1,L}$, $\rho_{2,L}$, $\rho_{1,N}$, $p_{1,H}$, $p_{1,L}$, $p_{2,L}$, $p_{1,N}$, $q_{1,H}$, $q_{1,L}$, $q_{2,L}$, $q_{1,N}$, $\vartheta_{1,H}$, $\vartheta_{1,L}$, $\vartheta_{2,L}$, $\vartheta_{1,N}$, $\varphi_{1,N}$, and $\varphi_{2,L}$, and is closed by conditions (10). The values of τ_H and λ are determined from (11) with allowance for the equalities $q_{1,M} = q_{1,H}$ and $\vartheta_{1,M} = \vartheta_{1,H}$. Calculations show that this regime, called the second irregular regime (IR2), exists at $a = 2$ for $93^\circ \le \varphi < 100.1^\circ$ ($\varphi_t = 120^\circ$) and at $a = 5$ ($\varphi_t = 101.54^\circ$) $78.3^\circ \le \varphi < \varphi_t$. In the region $1.9 < a < 3$ at angles of about 100° the IR2 is replaced by the third irregular regime (IR3) (Fig. 8), when the SC does not experience rotation, by intersecting a Mach wave (MW) at the point where the tangent to the MW makes the angle $\pi/2$ with the SC. The system of equations remains unchanged; however, in (11) λ should be replaced by τ_L since here we are concerned with the angle of rotation of the lower tangential discontinuity behind point O'' (Fig. 8). This regime exists up to $\varphi = \varphi_t$ ($100.1^\circ \le \varphi \le 120^\circ$ for $a = 2$). For interaction of two SSW the angle φ_t has the same meaning as the angle of complete refraction upon SSW incidence onto the interface of two media [1] and as soon as φ exceeds φ_t, the orientation of the MW curvature changes.

At $\varphi > \varphi_t$ for $a > 3$ the fourth irregular regime (IR4) is accomplished (Fig. 9a), which differs from the IR2 by orientation of an MW. It exists up to $\varphi = \pi$ since the angle $\varphi_{1,L}$ decreases monotonically (see Fig. 2). The system of equations does not change with the exception that $\varphi_{1,H}$ must be replaced everywhere by $\alpha = \pi - \varphi_H$. A slightly more complicated behavior of a shock-wave configuration is manifested at $1.9 < a \le 3$. At first here the fifth irregular regime (IR5) (Fig. 9b) is implemented (it exists for $120^\circ \le \varphi < 133.9^\circ$ at $a = 2$), which is rather broad for a close to 1.9 but then rapidly narrows with increasing a. Further increase of φ (and correspondingly decrease of $\varphi_{1,L}$) results in a smooth transition of the IR5 to the IR4, which, as in the previous case, is preserved up to $\varphi = \pi$.

Irregular Regimes of Interaction for Moderate a. In the range of a ($1.1 < a < 1.9$) with an increase of φ ($\varphi_{1,H} < \varphi_c$) the IR5 is replaced by the sixth irregular regime (IR6) (Fig. 10), where both triple points (O' and O'') depart from the SC that intersects an MW at the point O and the tangent to the MW makes an angle of $\pi/2$ with the SC. For air, krypton, and methane this occurs at $\varphi_{1,L} \approx 60^\circ$ ($\varphi_{1,H} < \varphi_c$). This regime is described by system (5) where $j = H, L, M, N$ and

$$i = \begin{cases} 1,\ 2, & j = H, L\ , \\ 1, & j = M, N\ . \end{cases} \tag{12}$$

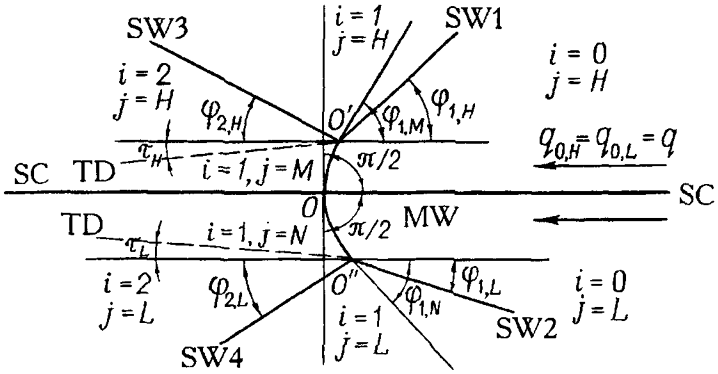


Fig. 10. Scheme of interaction of SSW in the IR6.

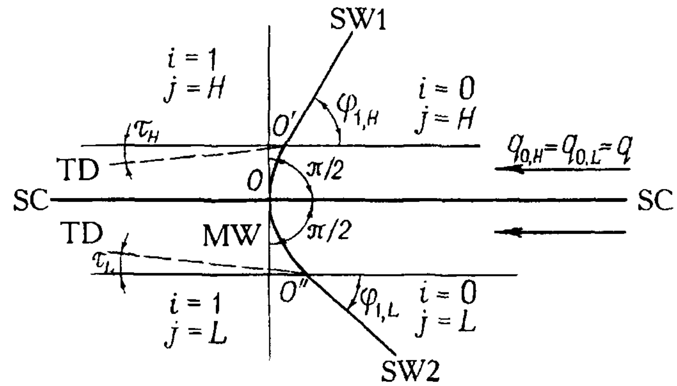


Fig. 11. Scheme of interaction of SSW in the TR7.

As for the IR4, here the TD is formed behind points O' and O'' with the angles of rotation respectively

$$\tau_H = \arctan \frac{q_{1,M} \sin \vartheta_{1,M}}{q}, \quad \tau_L = \arctan \frac{q_{1,N} \sin \vartheta_{1,N}}{q}. \tag{13}$$

System (5) and the conditions along the SC are the same as for the IR5.

The IR6 exists till $\varphi_{1,H} = \varphi_c$, which for $a = 1.333$ indicates the range $\varphi \sim 59.9 - 109.1^{\circ}$ ($\varphi_t \approx 138.59^{\circ}$), and then it is replaced by the IR5 (Fig. 9b), the further transformations of which with increasing φ (and decreasing $\varphi_{1,L}$) proceed in the same way as described in the previous paragraph for $1.9 < a < 3$.

Irregular Regimes of Interaction for Small a. In the range $1 < a \leq 1.1$ the IR1 (the IR1 is the initial regime here since an RCR is impossible) is replaced, as in the previous case, with increasing φ by the IR6 (Fig. 10), which exists till $\varphi_{1,H} = \varphi_c$ (for $a = 1.031$ φ ranges from $\sim 47.9^{\circ}$ to ~ 128.6), whereupon the IR3 is implemented (at $a = 1.031$, $\varphi_t \approx 165.93^{\circ} - 128.6^{\circ} < \varphi \leq 136.4^{\circ}$, with decreasing α the range rapidly narrows). The critical point for the IR3 is the instant when $\varphi_{1,L} = \varphi_c$, after which with φ increasing further the gas flow behind SW2 becomes subsonic and the seventh irregular regime (IR7) occurs (Fig. 11), which is described by system (5) at $i = 1$ and $j = H$, L. The regimes H and L at points O' and O'' coincide with M and N, respectively, the number of equations (8) corresponds to the number of variables: $\rho_{1,H}$, $\rho_{1,L}$, $p_{1,H}$, $p_{1,L}$, $q_{1,H}$, $q_{1,L}$, $\vartheta_{1,H}$, and $\vartheta_{1,L}$, and

$$\tau_H = \arctan \frac{q_{1,H} \sin \vartheta_{1,H}}{q}, \quad \tau_L = \arctan \frac{q_{1,L} \sin \vartheta_{1,L}}{q}. \tag{14}$$

The IR7 determines a shock-wave configuration up to $\varphi = \varphi_t$ ($\varphi_t = 165.93^{\circ}$ for $a = 1.031$). At this point the orientation of the curvature of the MW changes and, correspondingly, the IR7 is replaced by the eighth irregular regime (IR8) of interaction (Fig. 12). The system of equations remains unchanged with the exception that $\varphi_{1,H}$ must be replaced everywhere, as in the IR4, by $\alpha = \pi - \varphi_{1,H}$, and the values of τ_L and λ are determined from (14), where λ must be used instead of τ_H. The further evolution of the configuration with increasing φ is dictated by monotonically decreasing $\varphi_{1,L}$. As a consequence, the point $\varphi_{1,L} = \varphi_c$ is passed again but already in the direction of decreasing

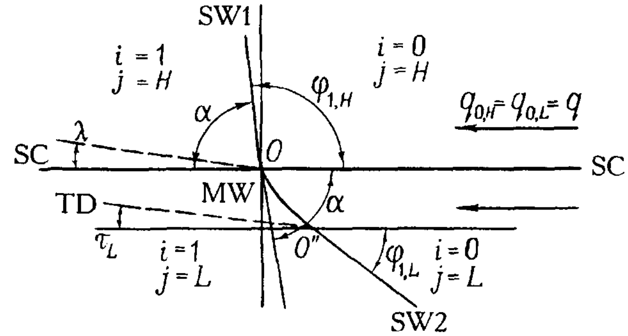


Fig. 12. Scheme of interaction of SSW in the IR8.

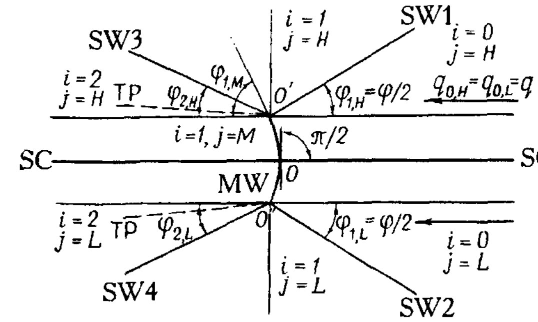


Fig. 13. Scheme of the RSA for identical SSWs.

angle $\varphi_{1,L}$, which gives rise to the IR5 (it exists at $a = 1.031$ for $175.6 \le \varphi < 179.1^{\circ}$). At $\varphi \approx 179.1^{\circ}$ the IR5 passes to the IR4, taking place up to $\varphi = 180^{\circ}$.

Thus, the approach suggested makes it possible to analyze evolution of a shock-wave configuration with increasing φ from 0 to 180°. Since the characteristic angles φ_R, φ_c, and φ_t are practically independent of particular values of D_H and D_L but depend on their ratio denoted by a and for the gases investigated $\varphi_R < 2\varphi_c < \pi$ ($\varphi_{1,H} = \pi/2$ at $\varphi = \varphi_t$) the relations given are valid for each of them.

Interaction of Two Weak SSWs. Since it is typical for weak SWs that $D_H \sim c$ and $D_L \sim c$ (at $D_H > D_L > c$) this type of interaction pertains to the region of small a. Here, a flow behind the SSW front is always subsonic; therefore configurations with triple points, like the RCR, cannot be obtained since $\varphi_R \to 0$ for $a \to 1$ [2]. At $a \to 1$ $\varphi_t \to \pi$, i.e., for weak shock waves the orientation of an MW cannot be changed and, consequently, only one regime, i.e., the IR7 (Fig. 11), is achieved.

On interaction of strong and weak SSW, $D_H \gg c$ for the strong SSW and $D_L \sim c$ ($D_L > c$) for the weak ones. This case pertains to very large a ($a > 3$) and is already considered in the respective paragraph.

Interaction of Identical SSW. This type of interaction corresponds to the limiting case $a = 1$; here

$$\varphi_H = \varphi_L = \frac{\varphi}{2}, \tag{15}$$

and $\varphi_t = \arccos(-1) = \pi$. The configuration branches H and L become absolutely identical. The dependence $\varphi_R(a)$ reveals a fast decrease in φ_R and vanishing of the latter near the point $a = 1$ [2]; consequently, the RCR is not implemented here. Analyzing system (5) in this case ($i = 1, 2$ for $j = H$ and $i = 1$ for $j = H, M$) with the conditions of equality of the pressure and the normal velocity component of the flow on each side of the SC (with a symmetrical configuration it is possible only when an MW intersects the SC at a right angle), we find, provided (9) is fulfilled, all parameters of the flow for this regime, called here the regime of small angles (RSA) (Fig. 13). The RSA exists as long as the condition

$$\cos\varphi_{2,H} \ge \sqrt{\left(\frac{1}{1 + \tan^2(\varphi_{2,H} + \vartheta_{1,H})\, K_{2,H}^2}\right)}, \tag{16}$$

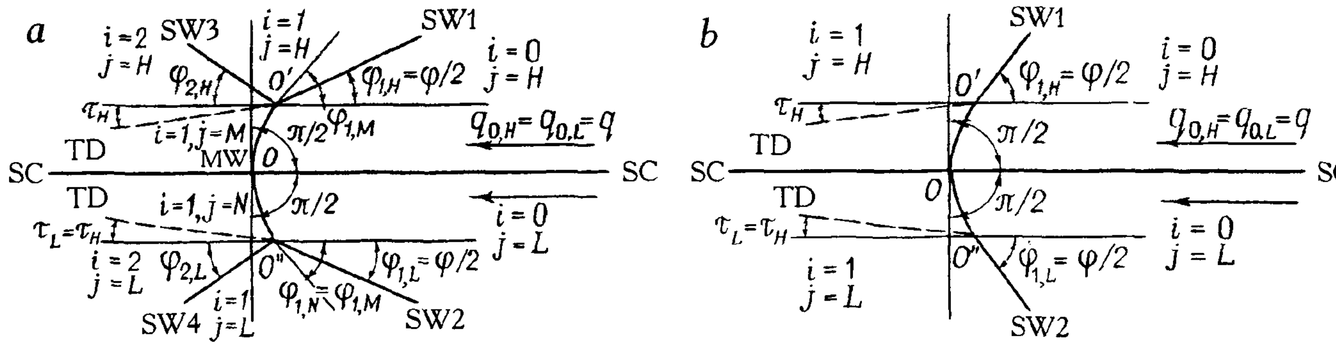


Fig. 14. Sixth (a) and seventh (b) regimes of irregular interaction for identical SSW.

is fulfilled. The equality sign in (16) (the $\varphi_{1,H}$ value appropriate for it is expressed in terms of ω) corresponds to the case when the MW becomes planar ($\varphi_{1,M} = \pi/2$) and the flow behind the configuration front is parallel everywhere to the SC (for air at $D_H = D_L = 1000$ m/sec, $\omega = 44.5^{\circ}$). A nonsymmetrical analog of the RSA also exists for $a \neq 1$, especially for small a and is a part of the IR5 at $\varphi < \omega$. Within the range of $2\omega \leq \varphi < 2\varphi_c$ a symmetrical analog of the IR6 is implemented (Fig. 14a). The calculation has already been considered above; the difference lies in the fact that the branches N and L coincide with the branches M and H, respectively, which results in a twofold decrease of the amount of equations and variables, $\tau_H = \tau_L$. With further increase in $\varphi \leq 2\varphi_c$ the IR6 is replaced by a symmetrical analog of the IR7 (Fig. 14b) since with increasing φ the branches H and L run through φ_c simultaneously. This regime exists till $\varphi = \pi$.

For the interaction of identical weak waves only one shock-wave configuration exists, i.e., the IR7 (Fig. 14b). It is calculated in the same manner as earlier with the only difference being that the branches H and L coincide and the number of equations decreases by a factor of 2.

Conclusion. This study has been restricted to obtaining general correlations for evolution of a shock-wave configuration while increasing the angle of contact φ of two planar SSW and establishing the characteristic angles. Here, as in [1], it has been assumed that the MW curve is fair and can always be described with a high degree of accuracy by the circumferential arc located between the bounding angles $\varphi_{1,M}$ and $\varphi_{1,N}$ (or $\varphi_{1,H}$ and $\varphi_{1,L}$ when the corresponding triple points are absent). The smooth change in the angle along a MW allows determination of characteristics of the flow behind it and of all hydrodynamic parameters describing the gas flow.

As follows from the foregoing, on interaction of two SSW a shock-wave configuration is determined by the quantity a, which has every reason to be called a parameter of the process, and by the relationship of the angle of contact φ of the SSW and three characteristic angles φ_R, φ_c, and φ_t, between which a definite relation always exists for gases ($\varphi_R < \varphi_c < \varphi_t$). Here, two of them ($\varphi_R$ and φ_t) are constants of the process. Knowing a priori these five quantities (a, φ, φ_R, φ_c, and φ_t), we can construct a priori the configuration formed upon interaction of two SSW.

It should be noted that similar calculations have been made by the present author for materials determined by a shock adiabat of the form $D = a_0 + bU$ (the system of equations corresponding to (5) is given in the Appendix). Calculations were made for water and some metals; general evolution of a shock-wave configuration remained the previous one. For gas media, similar calculations were made for air ($\rho = 1.29$ kg/m^3, $k = 1.4$), nitrogen (1.2506 and 1.4), argon (1.7837 and 5/3), hydrogen (0.0899 and 1.4), helium (0.1785 and 5/3), krypton (3.733 and 5/3), methane (0.717 and 4/3), neon (0.9004 and 5/3), and carbon dioxide (1.9724 and 4/3). They have demonstrated good agreement of the results with a very insignificant difference in the numerical values of the limiting angles characterizing the change of the regimes of interaction.

In conclusion, the author expresses his deep gratitute to G. S. Romanov for all kinds of support and continuing interest in the present work.

Appendix. When a shock adiabat of the form $D = a_0 + bU$ is used to describe a substance in which SSW interact, an analog of system (5), with all symbols adopted above preserved, is as follows:

$$p_{i,j} = p_{i-1,j} + \frac{\rho_{i-1,j} q_{i-1,j}^2}{b} \sin^2 (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j}) \left[1 - \frac{a_0}{q_{i-1,j} \sin (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})}\right],$$

$$\rho_{i,j} = \rho_{i-1,j} \frac{b}{b - 1 + \dfrac{a_0}{q_{i-1,j} \sin (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})}},$$

$$q_{i,j} = q_{i-1,j} \cos (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j}) \times$$

$$\times \sqrt{\left(1 + \frac{\tan^2 (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})}{b^2} \left(b - 1 + \frac{a_0}{q_{i-1,j} \sin (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})}\right)^2\right)},$$

$$\vartheta_{i,j} = \varphi_{i,j} - \arccos \sqrt{\left(\frac{1}{1 + \dfrac{\tan^2 (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})}{b^2} \left(b - 1 + \dfrac{a_0}{q_{i-1,j} \sin (\varphi_{i,j} + s_{i,j}\vartheta_{i-1,j})}\right)^2}\right)},$$

$$s_{i,j} = \begin{cases} 1, & i = 2 , \\ 0, & i \neq 1 , \end{cases}$$

The corresponding equations of equilibrium remain unchanged.

NOTATION

a, ratio of the velocity of the heavier shock wave to that of the lighter one ($a \geq 1$); D, shock-wave velocity; c, velocity of sound in the medium; p, pressure; q, total velocity of the flow in a coordinate system connected with the point of intersection of SSW; k, polytropic index of the gas; a_0 and b, coefficients of the $D-U$ adiabat; ρ, density; α, angle reverse to the slope to the SC of the stronger SSW; φ, angle of contact of the SSW; λ, angle of rotation of the SC behind the point O; τ, angle of rotation of the tangential discontinuity; ϑ, angle of rotation of the total velocity vector of the flow behind the shock wave front; ω, limiting angle of the RSA. Subscripts: H, upper part of the plane from the point O' and upward (or O if O' is absent in a drawing); L, lower part of the plane from the point O'' and downward (or O if O'' is absent in a drawing); M, region of the flow adjacent to the upper point O' of the MW; N, region of the flow adjacent to the lower point O'' of the MW; i, ordinal number of the discontinuity surface, its possible values are 0, 1, and 2; j takes values of H, L, M, and N; R indicates the limiting angle of the regime of regular collision; c, critical angle of transition of the flow behind the interacting SSW from the supersonic to subsonic mode; t, angle of the change of the spatial orientation of the MW determined from the value of arccos $(-1/a)$.

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